DO NOW

Recall the derivative formulas for inverse trig functions.

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$

$\frac{d}{dx}[\text{arccot}\, u] = -\frac{u'}{1+u^2}$

$\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}}$

$\frac{d}{dx}[\text{arccsc}\, u] = -\frac{u'}{|u|\sqrt{u^2-1}}$

5.8 Inverse Trigonometric Functions: Integration

The derivatives of the six inverse trig functions fall into three pairs. In each pair of cofunctions, the derivative of one function is the negative of the other.

Ex: $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\arccos u] = -\frac{u'}{\sqrt{1-u^2}}$

*** We only need to use the function of each pair... not the cofunction.

Theorem:

Let u be a differentiable function of x, and $a > 0$ and a constant.

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$
2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan \frac{u}{a} + C$
3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\,\text{arcsec}\, \frac{|u|}{a} + C$

***Rewrite the integrand into one of the above forms using u-substitution and/or pattern recognition.

Examples:

1. $\int \frac{dx}{\sqrt{49 - x^2}}$

$u = x$
$du = dx$
$a = 7$

$\int \frac{du}{\sqrt{a^2-u^2}}$

$\arcsin \frac{u}{a} + C$

$\boxed{\arcsin \frac{x}{7} + C}$

2. $\int \frac{dx}{2 + 9x^2}$

$u = 3x$
$du = 3dx$
$a = \sqrt{2}$

$\frac{1}{3}\int \frac{3\,dx}{2+9x^2}$

$\frac{1}{3}\int \frac{du}{a^2+u^2}$

$\frac{1}{3}\cdot\frac{1}{a}\arctan\frac{u}{a} + C$

$\frac{1}{3}\cdot\frac{1}{\sqrt{2}}\arctan\frac{3x}{\sqrt{2}} + C$

$\boxed{\frac{1}{3\sqrt{2}}\arctan\frac{3x}{\sqrt{2}} + C}$

3. $\int \frac{3dx}{25 + 16x^2}$

$u = 4x$
$du = 4dx$
$a = 5$

$\frac{3}{4}\int \frac{4dx}{25+16x^2}$

$\frac{3}{4}\int \frac{du}{a^2+u^2}$

$\frac{3}{4}\cdot\frac{1}{a}\arctan\frac{u}{a} + C$

$\frac{3}{4}\cdot\frac{1}{5}\arctan\frac{4x}{5} + C$

$\boxed{\frac{3}{20}\arctan\frac{4x}{5} + C}$

4. $\int \frac{dx}{x\sqrt{4x^2 - 9}}$

$u = 2x$
$du = 2dx$
$a = 3$

$\int \frac{2\,dx}{2x\sqrt{4x^2-9}}$

$\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

$\boxed{\frac{1}{3} \operatorname{arcsec} \frac{|2x|}{3} + C}$

5. $\int \frac{dx}{\sqrt{e^{2x}-1}}$

$u = e^x$
$du = e^x dx$
$a = 1$

$\int \frac{e^x\,dx}{e^x\sqrt{e^{2x}-1}}$

$\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

$\boxed{\operatorname{arcsec} |e^x| + C}$

6. $\int \frac{4x + 3}{\sqrt{1 - x^2}}\,dx$

$\int \frac{4x}{\sqrt{1-x^2}}\,dx + \int \frac{3}{\sqrt{1-x^2}}\,dx$

$u = 1 - x^2$
$du = -2x\,dx$

$u = x$
$du = dx$
$a = 1$

$\frac{4}{-2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx + 3\int \frac{du}{\sqrt{a^2-u^2}}$

$-2\int u^{-1/2}\,du + 3 \arcsin \frac{u}{a} + C$

$-2 \cdot 2u^{1/2} + 3 \arcsin x + C$

$\boxed{-4\sqrt{1-x^2} + 3\arcsin x + C}$

HOMEWORK

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